

ORBIT [Earth]

Fuel 72.1
Main 0.0
Howr 0.0

RCS
OFF ROT LIM

UT: Wed Mar 14 04:01:54 2001
MJD 51982.1680
Sim 3536s FoV 60°

Satellite Physics

Newton, orbits and Reclocking

December 2007
Morten Jagd Christensen

Transfer [Ref: Earth]

Src [self] Tgt Moon
TrL 341.14° TrL 232.11°
TLi N/A

HTO
SMa-0.01
TLi 116.16°
DTe 2.047k
Dv 107.3M

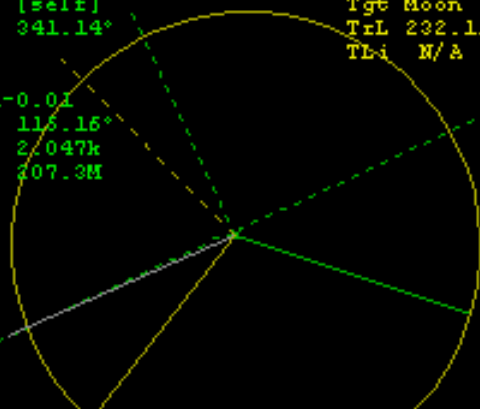
UPD

STP

EJ-

EJ+

DV



Orbit: Earth

From ECL	Prj SHP
--OSC.EL.--	---Moon---
SMa 6.739M	SMa 381.8M
SMi 6.739M	SMi 380.9M
PeR 6.723M	PeR 355.4M
ApR 6.755M	ApR 408.2M
Rad 6.750M	Rad 389.5M
Ecc 0.0025	Ecc 0.0692
T 6.505k	T 2.334M
PeT 2.072k	PeT 1.752M
ApT 4.824k	ApT 584.9k
Vel 7.677k	Vel 1.023k
Inc 5.08°	Inc 5.06°
LAN 102.54°	LAN 102.50°
LPe 116.63°	LPe 134.45°
AgP 14.29°	AgP 31.95°
TrA 224.31°	TrA 97.67°
TrL 341.14°	TrL 232.11°
MnA 224.31°	MnA 69.76°

REF

AR

TGT

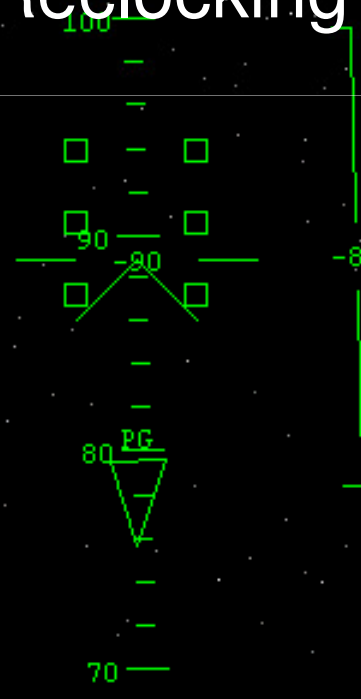
NT

MOD

PRJ

DST

HUD



Agenda

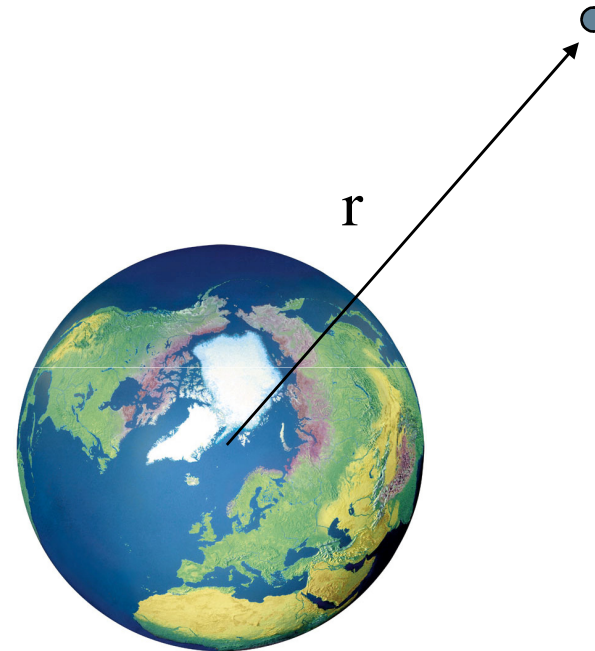
- Law of gravitation and of motion
- Geometry and dynamics of circular orbits
- Elliptical orbits
- Geosynchronous orbits
- Launching a satellite – Hohmann transfer
- Re-clocking
- Non-spherical mass distribution
- Deep space tracking and further complications
- Orbit adjustments
- DIY space travel with Orbiter

Newtons laws of motion

$$\bar{F} = m \cdot \bar{a} = m \cdot \frac{d\bar{V}}{dt}$$

$$\bar{F} = -\frac{GMm}{r^2} \hat{r}$$

$$\frac{d^2 \bar{r}(t)}{dt^2} = -\frac{GM}{r^2(t)} \hat{r}(t)$$



Differential equations (2D)

$$\frac{d}{dt} v_x(t) = -\frac{GM}{r^3(t)} x(t)$$

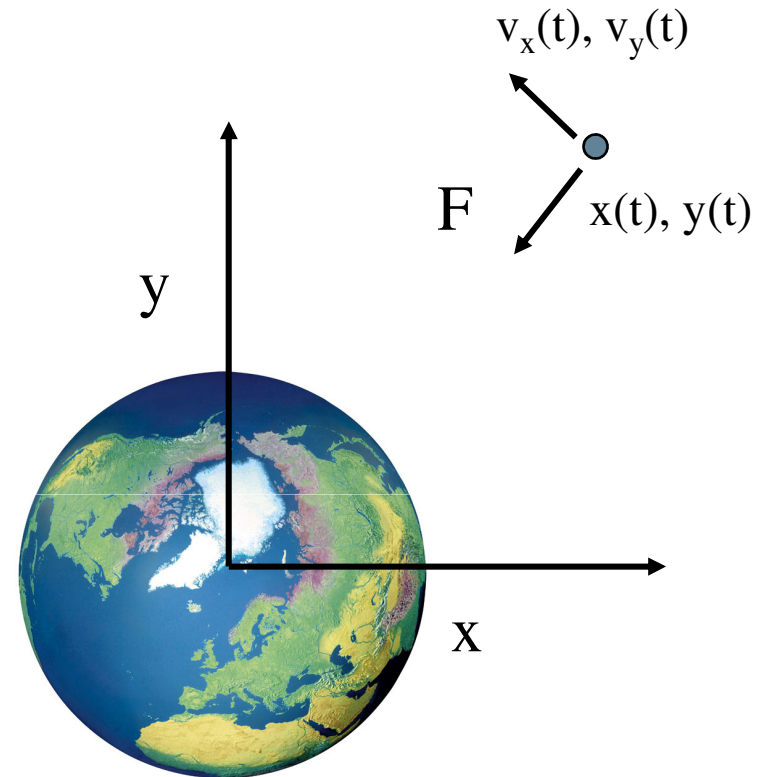
$$\frac{d}{dt} v_y(t) = -\frac{GM}{r^3(t)} y(t)$$

$$\frac{d}{dt} x(t) = v_x(t)$$

$$\frac{d}{dt} y(t) = v_y(t)$$

$$\bar{r}(t) = (x(t), y(t))$$

$$r(t) = \sqrt{x^2(t) + y^2(t)}$$



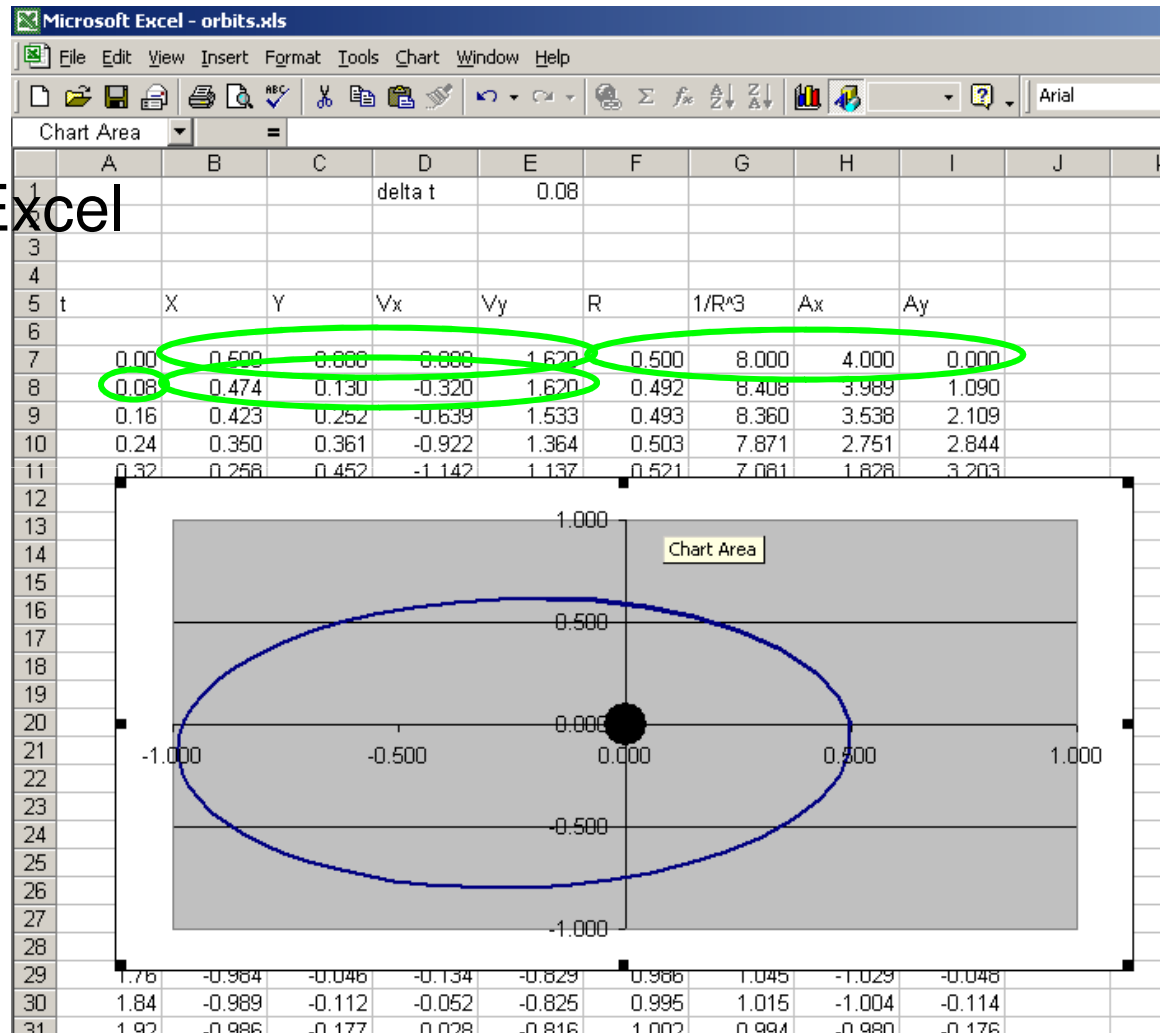
Solution: Ellipse

- Can be done in Excel

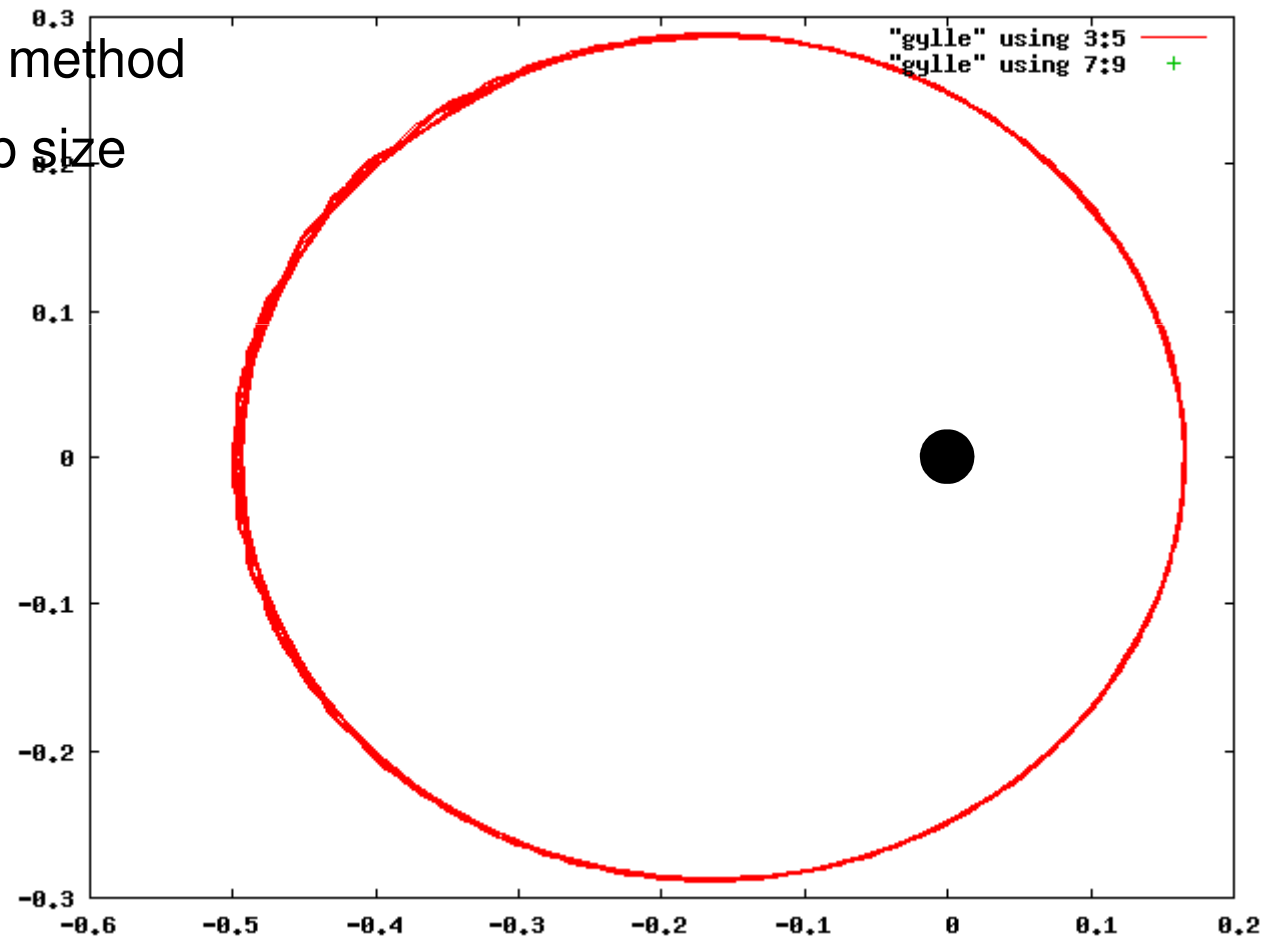
$$v_{x1} = v_{x0} + a_{x0} \cdot \Delta t$$

$$x_1 = x_0 + v_{x0} \cdot \Delta t$$

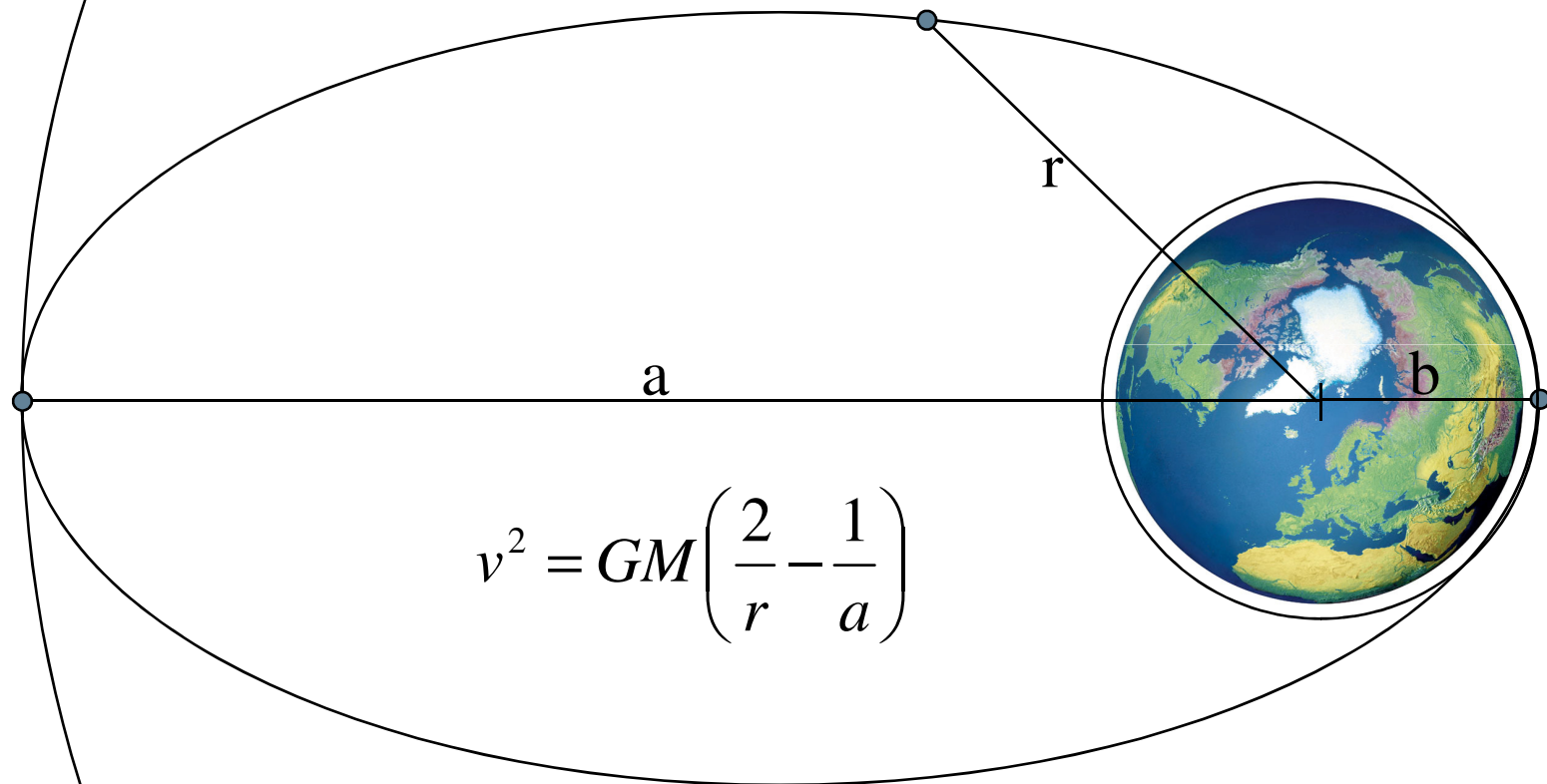
etc.



- Runge-Kutta method
- Adaptive step size
- Gnuplot



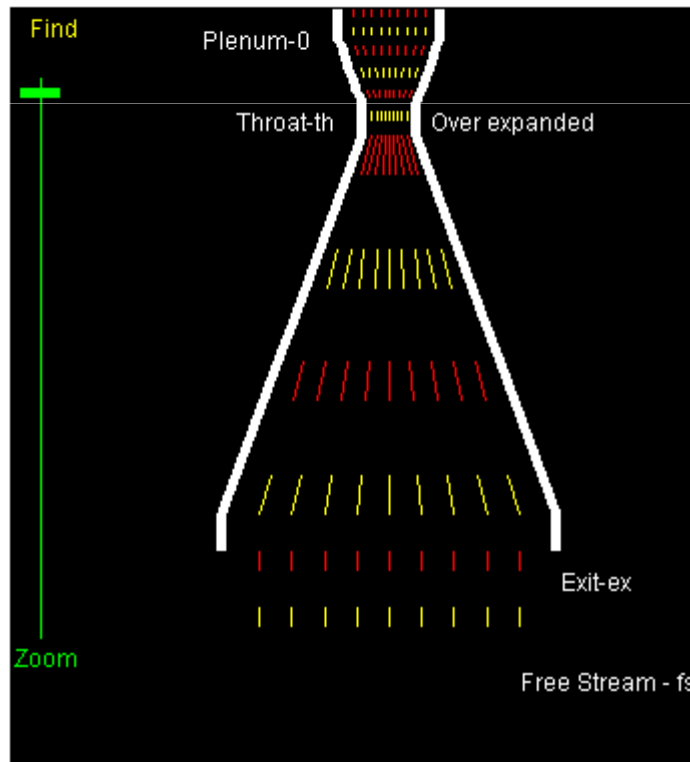
From low Earth orbit to GEO



Rocket Thrust

$$m \cdot \frac{\Delta V}{\Delta T} = F \Leftrightarrow \Delta T = \frac{m}{F} \cdot \Delta V$$

$$F = \dot{m}v_e + (p_e - p_0)A_e$$



Rocket Thrust Simulator Reset

Thrust **487612 lbs** Units: **English**

Flow **1532 pps** Isp **318 s**

Input Panel: **Geometry**

Ath sq in **161.7461** ◀ ▶

Aex/Ath **49.906** ◀ ▶

Ao/Ath **3.9925** ◀ ▶

Length ◀ ▶

Fuel H2	Oxidizer O2	O/F 8.0
Tt0 5870 F	Gamma 1.224	Mol Wt 16.0
Pt0 psi 2000.0	Pex 2.89	Pfs 14.7
Ath sq in 161.7	Aex 8072.1	NPR 136
Mth 1.0	Mex 4.541	Uex fps 12249

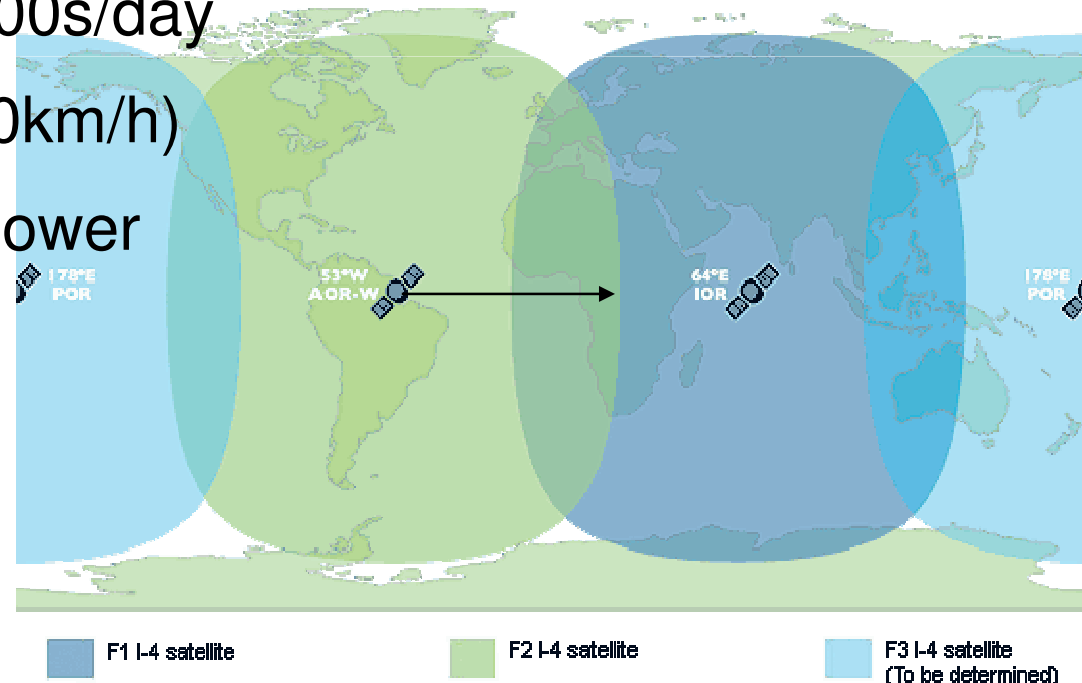
Burn times

- Centaur rocket GTO
 - Delta V for orbit insertion is ~ 2.4 km/s
 - Engines deliver ~ 147 kN
 - Mass: 15000kg prop + 6000kg sat
 - Burn time: ~ 343 s
- GTO to GEO
 - Delta V for circular orbit is ~ 1.5 km/s
 - Mass: 6000kg
 - Thrust data for I4 unavailable

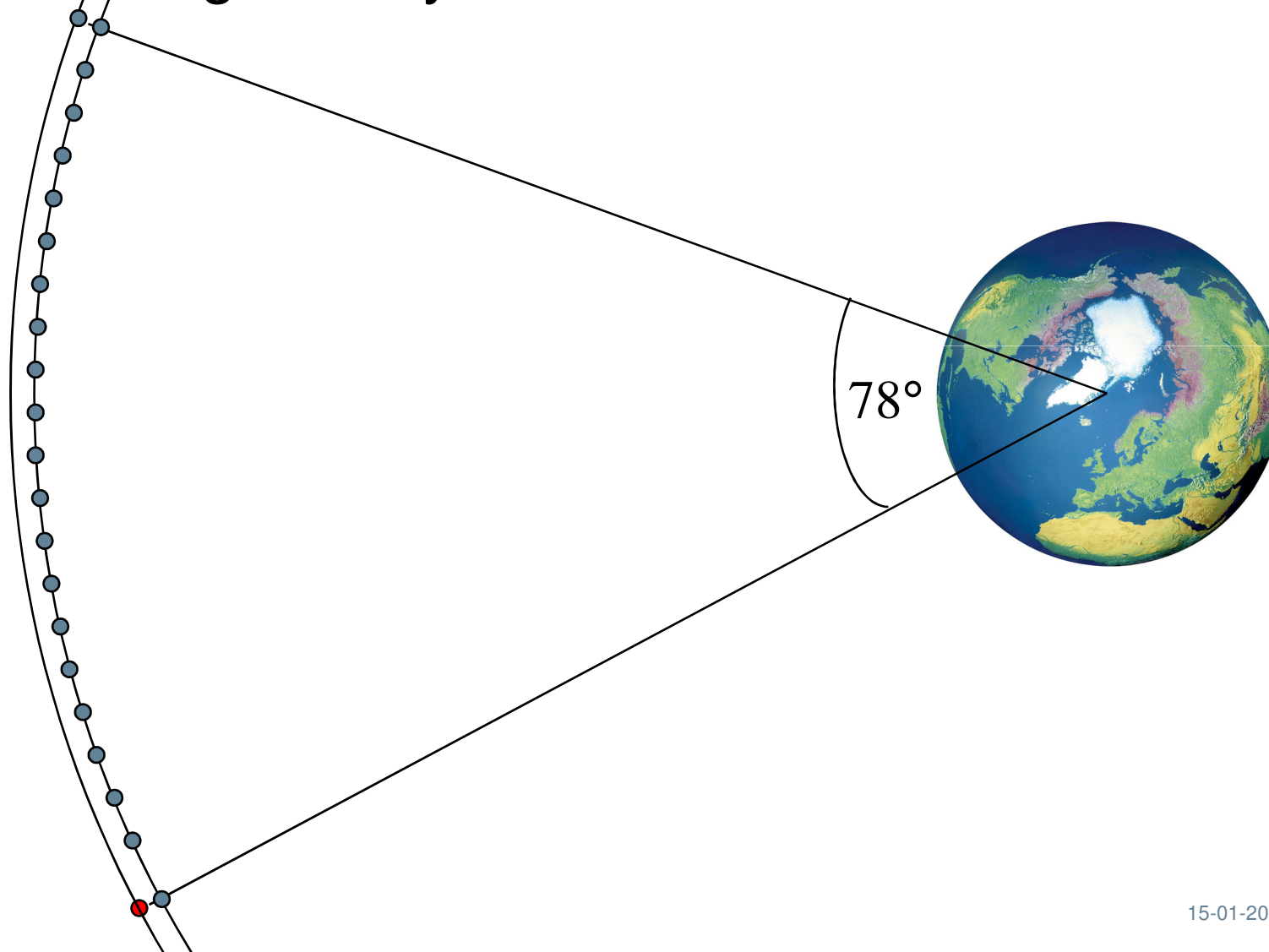


Re-clocking I4 satellites

- I4 F2 moves from 53W → 25E
- Distance travelled: $L = \frac{78^\circ}{360^\circ} 2\pi R$
- Time: 30days*86.400s/day
- Velocity ~23m/s (80km/h)
- New orbit is 650m lower

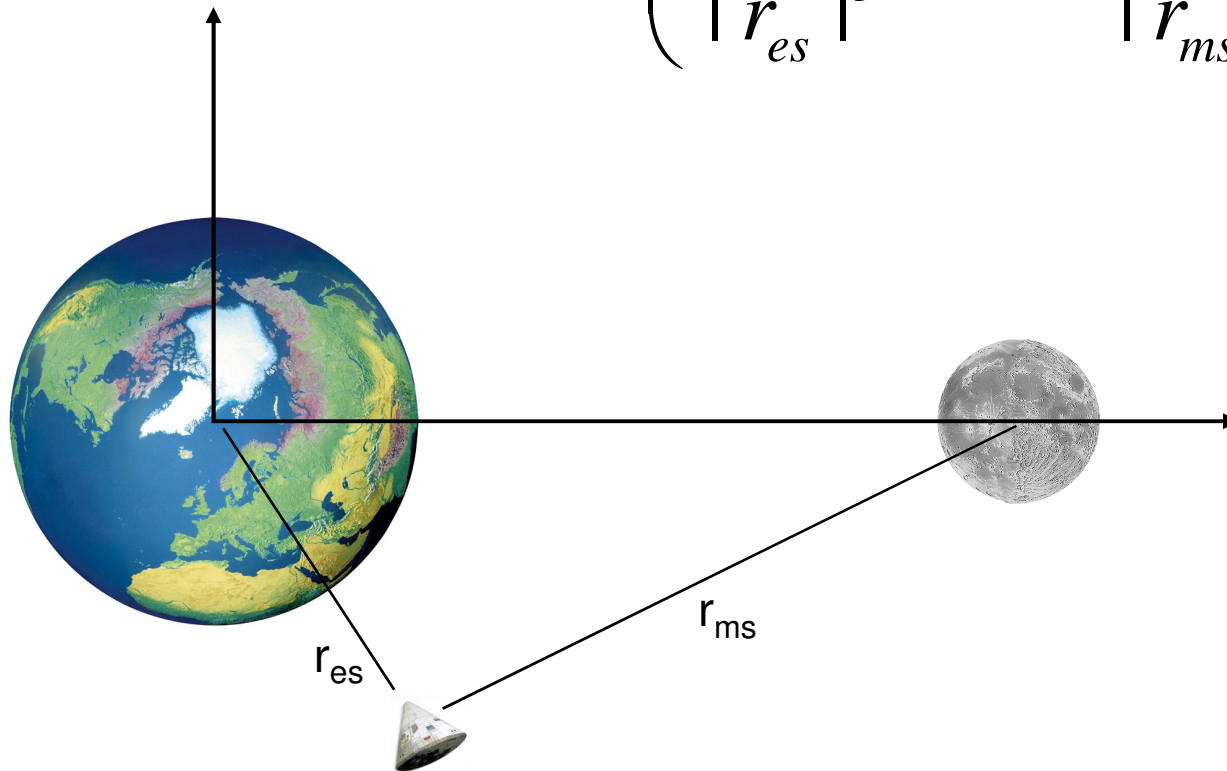


Re-clocking - finally

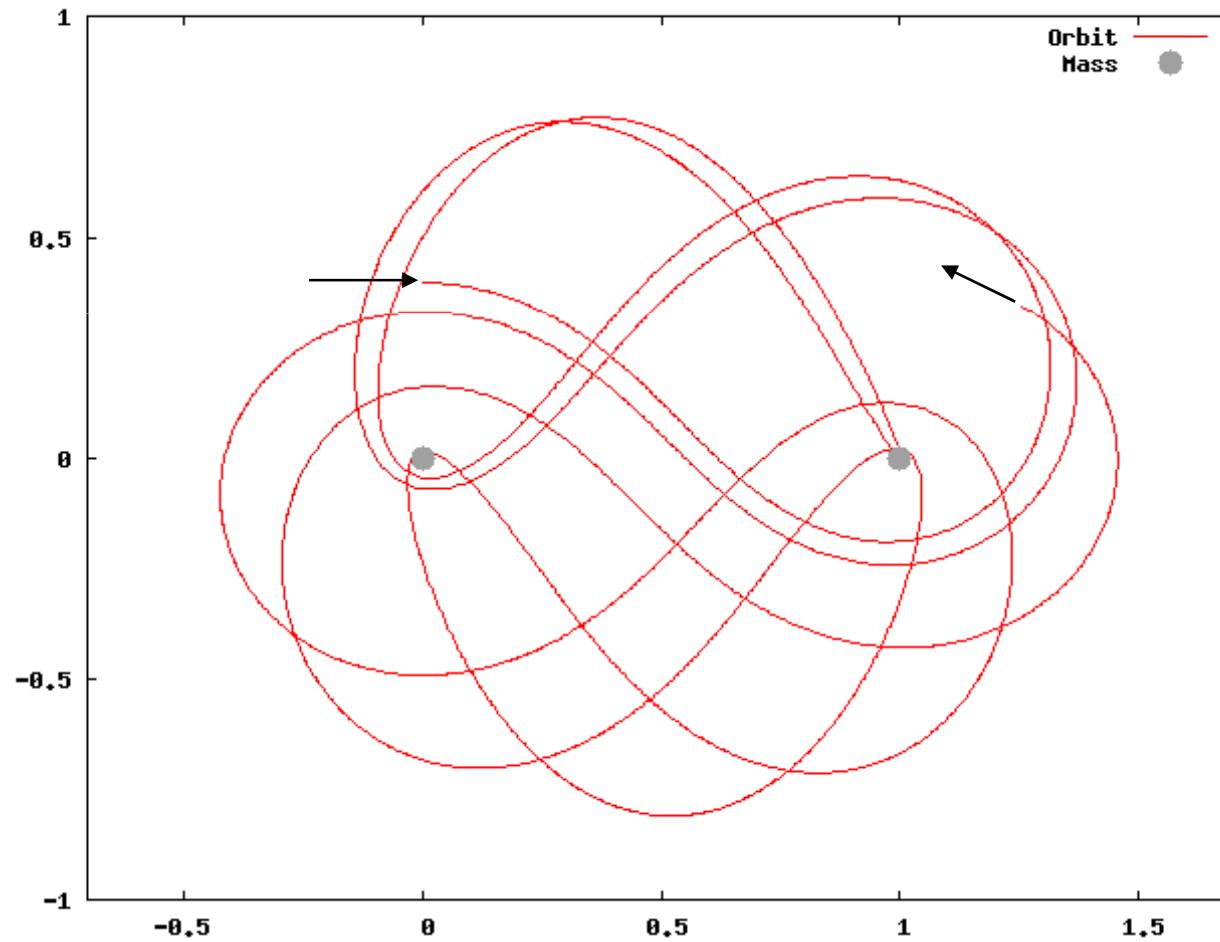


Three-body problem

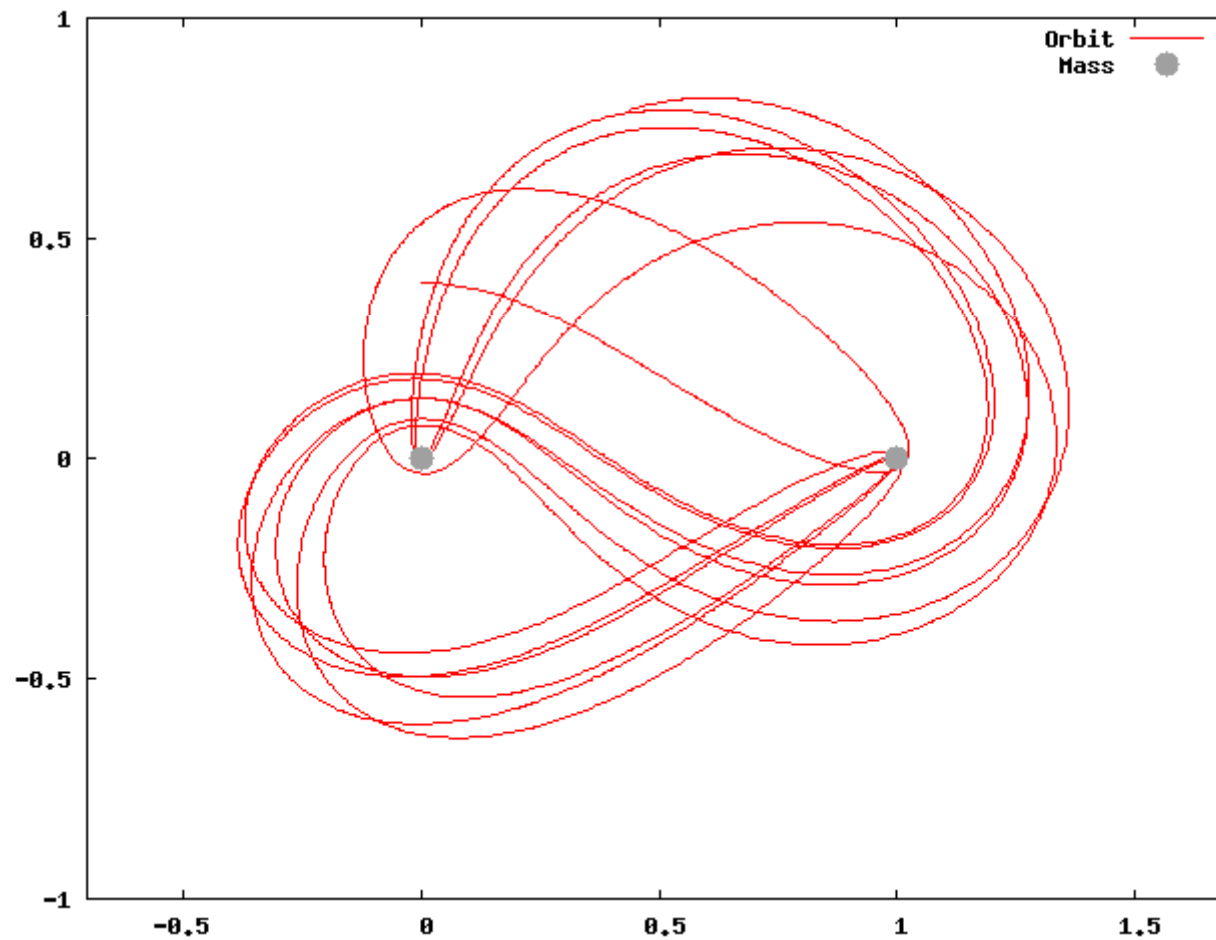
$$\bar{F}(\bar{r}) = -G \left(\frac{M_e m_s}{|\bar{r}_{es}|^3} \cdot \bar{r}_{es} + \frac{M_e m_s}{|\bar{r}_{ms}|^3} \cdot \bar{r}_{ms} \right)$$



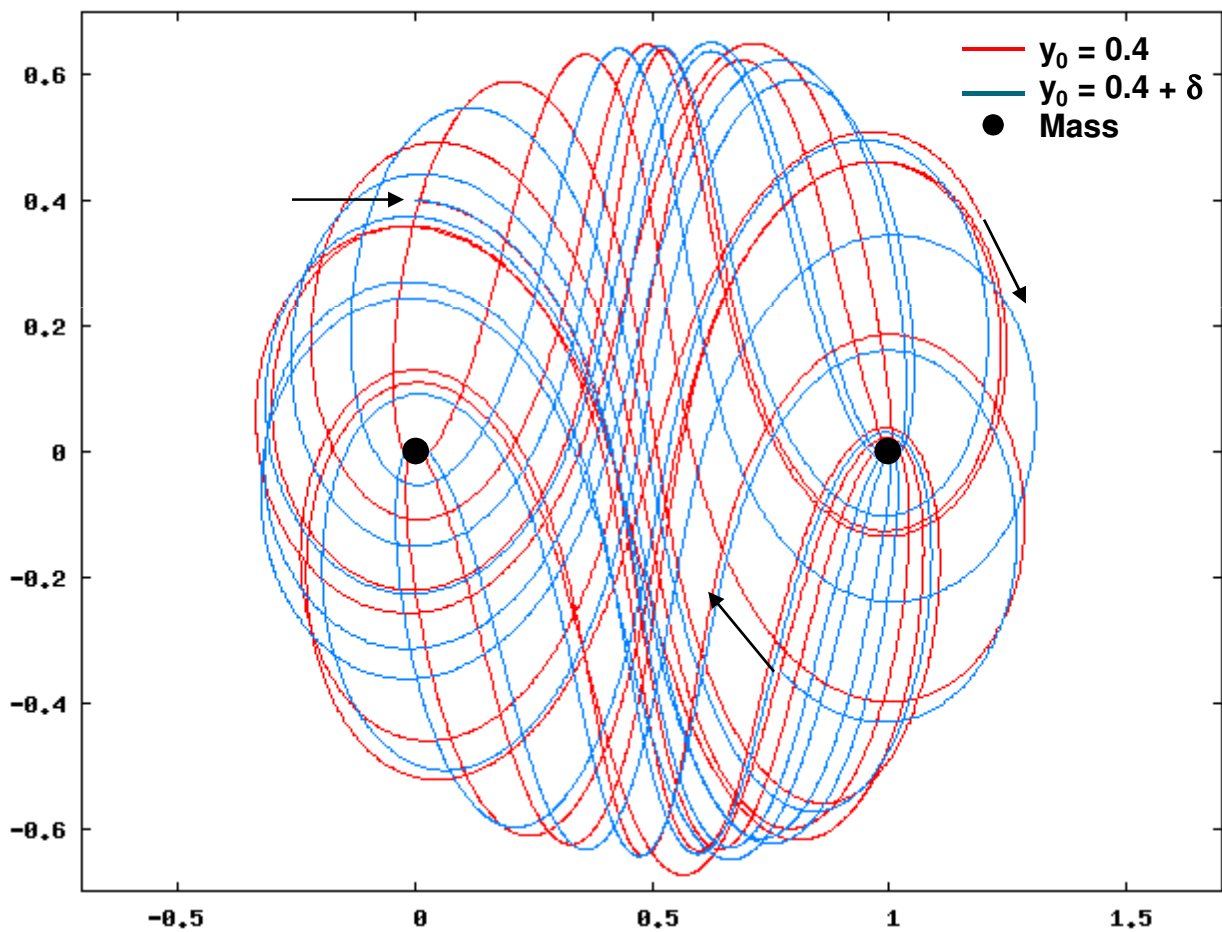
Chaotic orbits



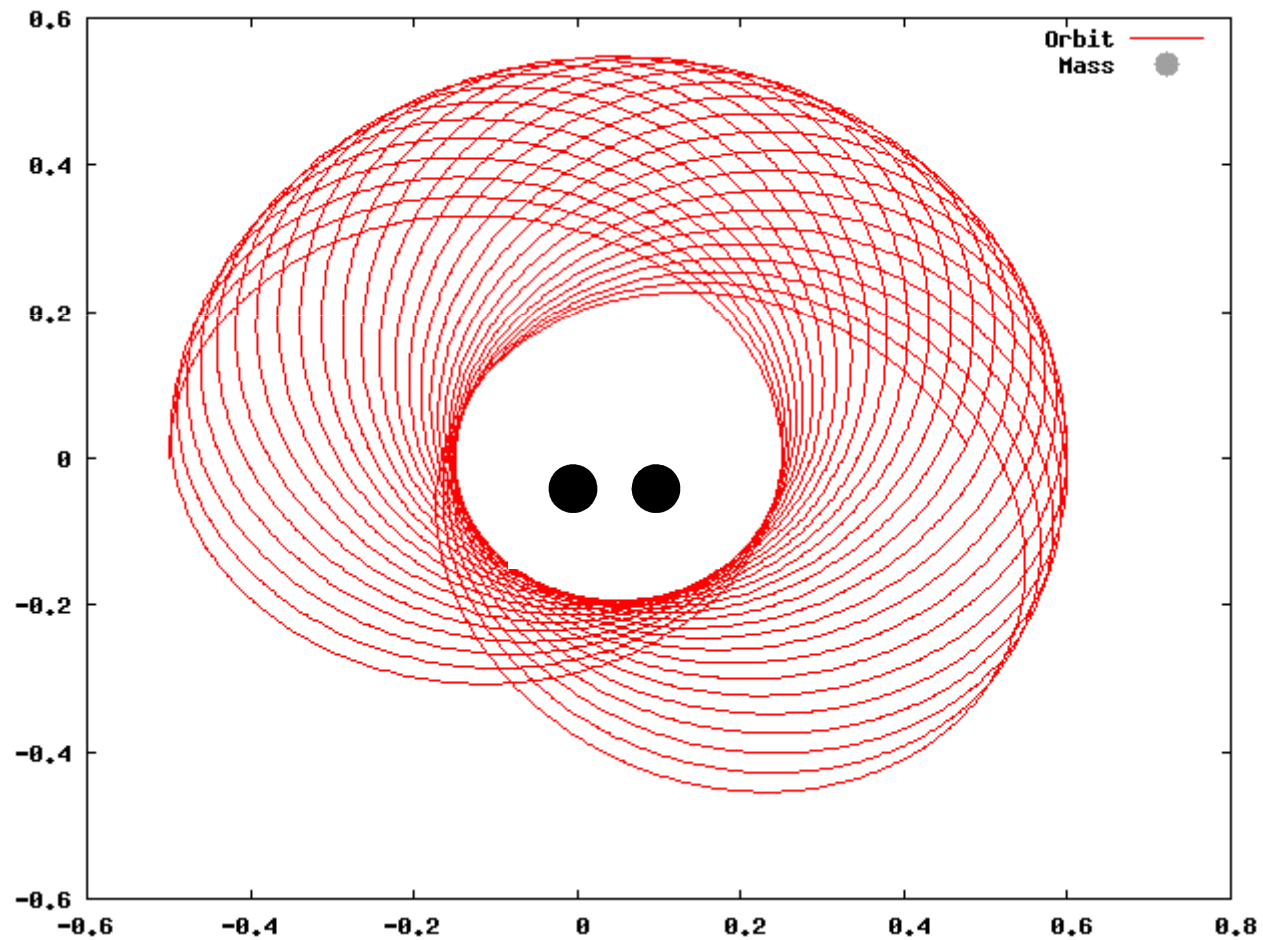
Chaotic orbits?



Yes really!



Non spherical mass distribution



Non-spherical gravitational field

$$U(r, \theta) = -\frac{GM}{r} \left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R}{r} \right)^n P_n(\sin(\theta)) \right]$$

Earth data

$$J_2 = 1082.63 \cdot 10^{-6}$$

$$J_3 = -2.51 \cdot 10^{-6}$$

$$J_4 = -1.60 \cdot 10^{-6}$$

$$J_5 = -0.15 \cdot 10^{-6}$$

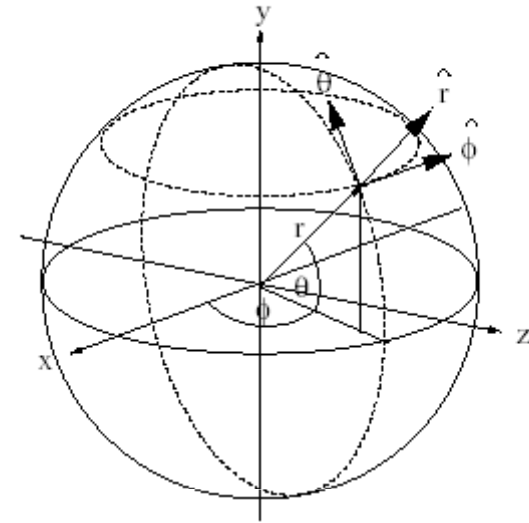
$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

...



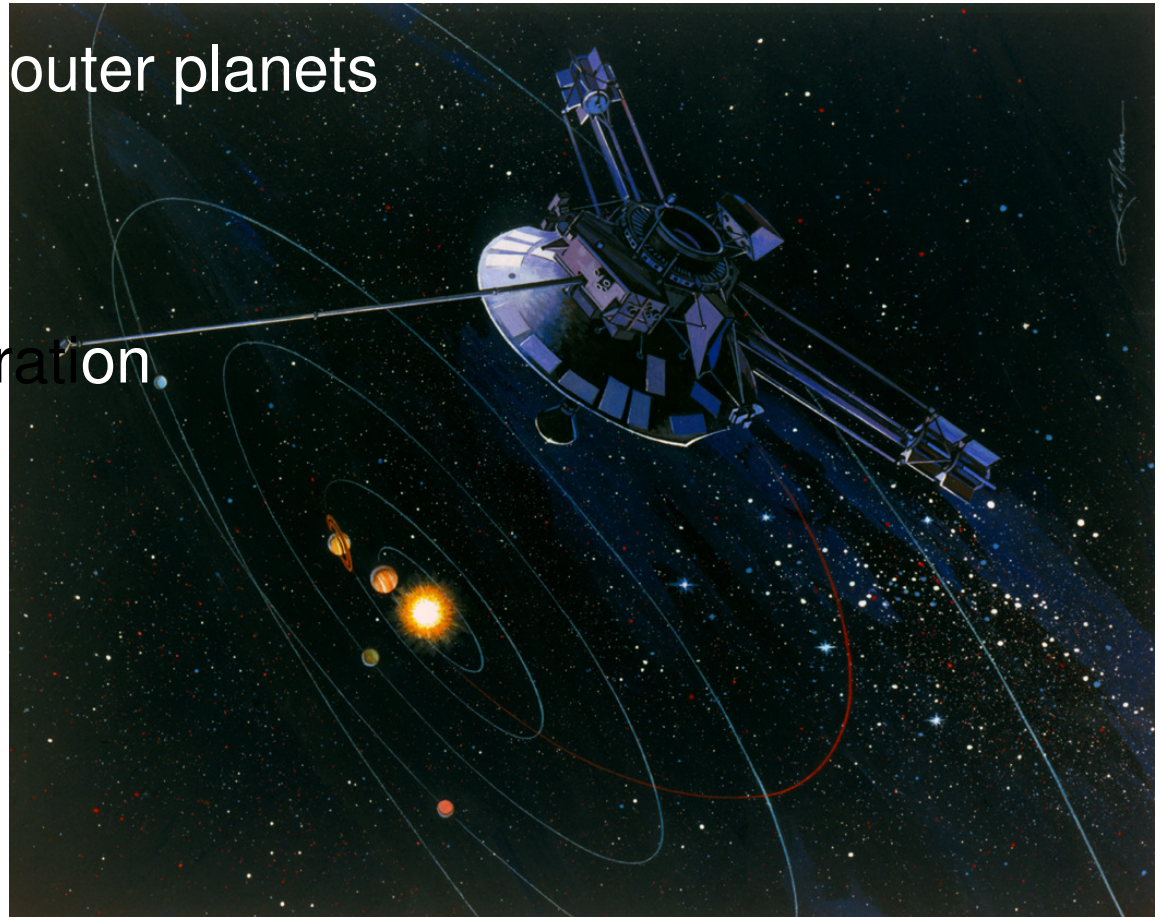
And it gets worse!

- Relativistic spacetime effects
- Earth geometry corrections
 - Tides
 - Plate tectonics
 - Pole movements
 - Internal mass shifts
- Pico second time resolution

$$\ddot{\mathbf{r}}_i = \sum_{j \neq i} \frac{\mu_j (\mathbf{r}_j - \mathbf{r}_i)}{r_{ij}^3} \left(1 - \frac{2(\beta + \gamma)}{c^2} \sum_{k \neq i} \frac{\mu_k}{r_{ik}} - \frac{2\beta - 1}{c^2} \sum_{k \neq j} \frac{\mu_k}{r_{jk}} - \frac{3}{2c^2} \left[\frac{(\mathbf{r}_j - \mathbf{r}_i) \dot{\mathbf{r}}_j}{r_{ij}} \right]^2 + \frac{1}{2c^2} (\mathbf{r}_j - \mathbf{r}_i) \ddot{\mathbf{r}}_j - \frac{2(1 + \gamma)}{c^2} \dot{\mathbf{r}}_i \dot{\mathbf{r}}_j + \right. \\ \left. + \gamma \left(\frac{v_i}{c} \right)^2 + (1 + \gamma) \left(\frac{v_j}{c} \right)^2 \right) + \frac{1}{c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}^3} \left([\mathbf{r}_i - \mathbf{r}_j] \cdot [(2 + 2\gamma) \dot{\mathbf{r}}_i - (1 + 2\gamma) \dot{\mathbf{r}}_j] \right) (\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_j) + \frac{3 + 4\gamma}{2c^2} \sum_{j \neq i} \frac{\mu_j \ddot{\mathbf{r}}_j}{r_{ij}} \quad (3)$$

Satellite tracking - Pioneer 10

- First probe to visit outer planets
- Tracking data from 1972 to 2002
- Anomalous acceleration.
 $8 \times 10^{-8} \text{G}$



Space travel with Orbiter

Top Left Fuel Status:

- Fuel: 100.0
- Main: 0.0
- Stor: 0.0
- RCS: OFF ROT LIN

Top Right: SIM 8400705 FOV 30

Transfer [Ref: Mars] Data:

Parameter	Value	Target (Deimos)
Src [self]		
TrL	246.43°	87.28°
HT0		TLi 205.58°
SMa	16.59M	
TLc	359.00°	
DTc	8.555k	
Dv	418.1	
TLi	167.39°	
DTi	35.93k	
RInc	1.94°	

Map: Deimos (TRK):

- Target base: N/A
- Target orbit: N/A

Control Panels:

- Left Panel:** REF, TGT, TRK, ZM, <<, >>
- Right Panel:** REF, SRC, TGT, NT, HTO, NUM, UPD, STP, EJ-, EJ+, DV-, DV+
- Bottom Panel:** PWR, SEL, MNU, KILL ROT, HOR LVL, PRO GRD, RETR GRD, NML +, NML -, HOLD ALT

Orbit Diagram: A circular diagram showing the transfer path from Mars (left) to Deimos (center). The path is a dashed line. Key angles are marked: -20, -30, 280, 40, 270, 290, and PG.

View srb1
Mode global frame
Dist 146.1

UT. Wed Mar 14 14:15:55 2001
MJD 51982.5944
Sim 5433s FoV 80°

- Full solar system dynamics simulation
 - Including non spherical gravitational fields
 - Sun, all planets, moons, and other orbiting objects
 - Timewarp for interplanetary cruises
 - Broad range of spacecraft
- Realism settings
 - Atmospheric haze, clouds
 - Aerodynamics in earths atmosphere
 - Rocket engine exhaust
- Launch from Earth, land on moon base
- Realistic on board flight software
- Star constellation overlay

Orbiter Demo

References

- **J.D. Anderson, et.al.** *"Study of the anomalous acceleration of Pioneer 10 and 11"*
- **A. Celetti et. al.** *"The Celestial Waltz"*
- **Inmarsat**
- **H.Kartunen, et. al.** *"Fundamental Astronomy"*
- **NASA** *"Basics of Space Flight"*, <http://www.jpl.nasa.gov/basics>
- **NASA** *"Beginner's Guide to Rockets"*
<http://exploration.grc.nasa.gov/education/rocket/bgmr.html>
- **M.Schweiger** *"Orbiter - Space Flight Simulator"*, <http://www.orbitersim.com>
- **W.H.Press, et. al.** *"Numerical Recipes in C"*, <http://www.nr.com>